Reliability Engineering and System Safety 94 (2009) 370–381

Contents lists available at ScienceDirect

Reliability Engineering and System Safety

journal homepage: www.elsevier.com/locate/ress

Maintenance and replacement policies under technological obsolescence

Julien Clavareau *,[1], Pierre-Etienne Labeau

Service de Métrologie Nucléaire, Université Libre de Bruxelles, Av. Franklin Roosevelt 50, CP165/84 Bruxelles B-1050, Belgium

ARTICLE INFO

Article history:
Received 16 October 2007
Received in revised form
25 February 2008
Accepted 29 March 2008
Available online 11 April 2008

Keywords:
Preventive and corrective maintenance
Replacement strategies
Technological obsolescence
Monte Carlo simulation

ABSTRACT

The technological obsolescence of a unit is characterized by the existence of challenger units displaying identical functionalities, but with higher performances. This paper aims to define and model in a realistic way, possible maintenance policies of a system including replacement strategies when one type of challenger unit is available. The comparison of these possible strategies is performed based on a Monte Carlo estimation of the costs they incur.

1. Introduction

Most often, papers studying optimization of preventive or corrective maintenance policies rely on the assumption that failed or used pieces of equipment are replaced by identical items. Actually, the technological reality is often quite different. In practice, new equipments are regularly available on the market achieving the same missions, but with higher performances. These higher performances can be understood as smaller failure rates, lower energy consumption, a lower purchase cost, etc. At the same time, it can be more and more difficult or costly to find old-generation spares to replace degraded units. This situation is characteristic of technological obsolescence.

Managers then face important issues, such as, for instance: how to optimally schedule the replacement of old-type units by new-type ones? Is it economically more interesting to preventively replace all the old equipments, without benefiting from their residual lifetime, by their more performing challengers, or on the contrary is it preferable to replace gradually the old components in a corrective way, progressively with their normal outage, but at the risk of a larger number of failures? Such questions become even more crucial when spare parts are to be dealt with.

The aim of our work is therefore to define replacement policies of these obsolete equipments and to help the decision maker find an optimal strategy among them.

Previous works envisaged this problem in a simplified way.

In Ref. [1], the case of one single component subject to ageing, which can be either periodically maintained or replaced by a technologically more advanced unit was studied.

In Ref. [2], authors studied analytically the following case: A set of n identical and independent units can be either preventively or correctively replaced by new-type units. The replacements take a negligible time. The new-type units have a lower constant failure rate and a lower consumption rate. The so-called "K strategy" was introduced as follows [3]: first, new-type components are used only to replace failed old-type units; then, after K corrective actions of this kind, the $n-K$ old-type remaining components are preventively replaced by new-type ones at the time of the Kth corrective intervention. The 0 strategy represents the preventive replacement of all old-type components at the initial moment. In Ref. [2], the authors reached the following conclusion: no matter which values are chosen for the data and the time horizon, only three strategies can be optimal: either all the components are replaced preventively ($K=0$), or one component is replaced correctively and the others preventively immediately after this first failure ($K=1$), or all the components are replaced correctively ($K=n$).

In Ref. [4], units subject to ageing and non-negligible stochastic replacement durations were envisaged and the same conclusions reached.

In the continuation of the works presented in Refs. [2,4], we introduce in this work more realistic maintenance actions, as extensions of the K strategy, and develop a complete model for the

* Corresponding author. Tel.: +32 2 650 40 42; fax: +32 2 650 45 34.
E-mail addresses: jclavare@ulb.ac.be (J. Clavareau), pelabeau@ulb.ac.be (P-E Labeau).

[1] Grant FRIA, Belgium.

0951-8320/$ - see front matter © 2008 Elsevier Ltd. All rights reserved.
doi:10.1016/j.ress.2008.03.033

management of a set of identical units subject to obsolescence, in the presence of a maintenance policy and of challenger units with a limited number of maintenance teams.

This paper summarizes and extends the works presented in Refs. [5,6]. It is organized as follows: Section 2 describes the model proposed and the assumptions on which it is based. Section 3 illustrates, by numerical results based on Monte Carlo (MC) simulation, some aspects of the whole model and some of the strategies proposed. In particular, we compare in Section 3.1 our MC results with the analytical solution of the simplified problem from [2]. Section 3.2 treats how to deal with the spare part inventory and the time horizon on which the transition between technological generations takes place. In Section 3.3, we discuss on the basis of another set of data the ability to forecast a budget for the replacements, which is regularly distributed in time. Finally we conclude by some possible perspectives and extensions of the model.

2. Model description

In this work, we will consider a set of n identical units, likely to be replaced by their more performing challengers. These units are subject to ageing and can be either replaced, imperfectly preventively maintained or repaired.

In this work, the new-type units will be more performing according to two criteria: their consumption rate and failure rate will be lower than that of the old ones.

2.1. Component failure modes

For both generations of components, we consider the following failure causes:

- *Ageing*: we model the ageing of the components by failure times exponentially distributed with time-dependent failure rates. These failure rates are the sum of a constant term λ_0 embodying purely random failures and a time-shifted Weibull-like contribution corresponding to ageing. At time t, the cumulative probability function of the failure time of one unit of a given generation is thus given by an expression of the form:

$$F(t)=\begin{cases}1-e^{-\lambda_0(t-t_s)} & \text{if} \quad \tau(t)\leqslant v\\ 1-e^{-\lambda_0(t-t_s)((\tau(t)-v)/\alpha)^\beta} & \text{if} \quad \tau(t)>v \text{ and } \tau(t_s)\leqslant v\\ 1-\dfrac{e^{-\lambda_0(t-t_s)((\tau(t)-v)/\alpha)^\beta}}{e^{-((\tau(t_s)-v)/\alpha)^\beta}} & \text{if} \quad \tau(t_s)>v\end{cases} \tag{1}$$

  where t_s is the instant at which the unit underwent the last intervention and $\hat{o}(t_s)$ the unit age after the intervention. The location parameter í is an effective working time before which the ageing of the unit does not affect its failure rate. The age $\hat{o}(t)$ to be considered to evaluate the failure probability is the effective (or virtual) age of the unit. This effective age is different from the calendar working time of the unit and depends on its past and on the maintenance interventions it has undergone. In particular, we consider that the different interventions affect this effective age by a rejuvenation factor. See Section 2.2, Eq. (3), for more details.
- *Common cause failures*: we also consider possible common cause failures modelled according to Atwood's shock model [7]. This model considers one (or several) external cause(s), whose occurrence entails an on-demand failure risk for the units, with a failure probability possibly specific to each component. In this work, the occurrence of the only initiating cause considered is distributed according to a negative exponential pdf (parameter χ); it is supposed that the old-type units have a conditional failure probability p_O different from that p_N of the new-type equipment.
- *On-demand start-up failure*: For the stocked units, we assume a cold stand-by situation. When a component is replaced by a new one, the latter has a probability to fail on demand depending on the storage time. In the model, this probability is given by a Weibull law of the form (1).
- *Incompatibility*: As already introduced in Ref. [4], a probability of incompatibility $p_{inc}(t)$ is accounted for, in order to model the fact that the on-site implementation of new-type components could turn out to be problematic, and some replacements could not be immediately successful, as technicians are not familiar yet with this new technology. This probability of incompatibility is an on-demand probability of unsuccessful restart after a replacement of an old-type unit by a new-type unit. For these replacements, the incompatibility comes in addition to the on-demand start-up failure.
- Part of this probability will decrease when both the informa tion on the installation procedure and experience increase. This incompatibility should consequently not favour early replacements. This incompatibility hazard is difficult to model, especially its time-dependent part. A first approach consists in limiting this dependence on the number of replacements per formed on the system under study. Adopting this simplification, we can write:

$$p_{inc}(t)=p_0+\frac{p_i}{f^{n_{int}(t)-1}} \tag{2}$$

  where p_0 is the purely ra ndom on-demand failure probability of the new-type components, p_i is the contribution to incompatibility for the first replacement intervention, $n_{int}(t)$ is the number of replacements of old-type units by new-type ones achieved on the whole system and f is a parameter larger than 1. Yet a more satisfying modelling depending on the calendar time should be developed in future works.

2.2. Interventions

As mentioned before, we consider a limited number of maintenance teams, which are supposed to perform four different types of actions. The first one is the preventive imperfect maintenance of the components at constant time intervals. The second one consists in repairing a failed component. The last two actions are the corrective and the preventive replacements.

To model the effects of these interventions, we use, among the available models in the literature, the effective age model [8,9]. This effective age is different from the physical component age (absolute or calendar age) that gives the time elapsed from the time the component was first started until the current time. The effective age rather represents an equivalent working time of the unit, given the different interventions it has undergone. The effective age $\tau(t)$ is the one taken into account (see Eq. (1)) in the Weibull law to evaluate the failure probability of a component.

The imperfect preventive maintenance actions are carried out at regular intervals. The effect of an imperfect preventive maintenance is to reduce the unit's effective age by a rejuvenation factor ε_m. The effective age τ_a after a preventive maintenance is given by

$$\tau_a=\varepsilon_m\tau_b, \quad 0\leqslant\varepsilon_m\leqslant 1 \tag{3}$$

where τ_b is the effective age of the components before the maintenance and ε_m ($\leqslant 1$ except if the intervention deteriorates the component) is the rejuvenation factor due to a preventive maintenance. If $\varepsilon_m=0$, the maintenance is perfect (*as good as new*) and if $\varepsilon_m=1$ the maintenance has no effect (*as bad as old*).

This use of the concept of effective age here above is referred to in different yet equivalent ways in the literature, e.g. Kijima 2 model [10], Arithmetic Reduction of Age with infinite memory [11] or to the Proportional Age Setback model [12].

We assume that no intervention is perfect; hence each maintenance operation may have a different impact. This implies that the rejuvenation factor ε_m should not be taken constant. Moreover, it is assumed to be a random variable embodying the variability in the resulting state of the component. The expected effect of a preventive maintenance is less and less efficient as the number of maintenance actions undergone by a component gets higher and the unit's work time gets higher. Thus the distribution of $\varepsilon_m(\tau_b)$ should also depend on the age of the component (or on the number of maintenance actions already performed).

In addition, we consider that the effective age of the component is a degradation criterion. A preventive replacement is planned when the effective age of the component reaches a threshold value, τ_{max}. This threshold value is fixed such that it becomes more expensive on average to maintain the component and replace it at the next maintenance time than to replace it immediately, given the expected costs induced by failures during the inter-maintenance period. In order to delay or advance the installation of the new-type units in the system, the threshold value for the old-type units can be augmented or decreased, respectively, during the transition period. There are thus three different threshold values to be considered: one for the replacement of old-type components by old-type spares, one for the replacement of old-type components by new-type ones and one for the replacement of new-type components by new-type spares.

When a failure occurs, if a preventive replacement was planned but not yet completed, the component is repaired; otherwise, the component is correctively replaced. After repair, a component can be in either a degraded or a rejuvenated state compared to its state before failure. The repairs are assumed to be, in average, minimum. It means that, in average again, the effective age of a component after a repair is the same as before failure. The effective age after repair is thus given by an equation similar to (3) but with an age reduction factor ε_r associated to a repair, whose expected value is equal to one.

In our model, the intervention durations are random variables modelled by Erlang distributions. We suppose that the preventive maintenance actions and the replacements are always successfully performed within a given intervention time interval (lower and upper bounds). As Erlang distributions have a non-bounded support, they are truncated in these two cases at these extreme intervention times. These bounds are noted D_{mmin} and D_{mmax} for the minimum and the maximum maintenance durations, respectively, and D_{rmin} and D_{rmax}, for the minimum and the maximum replacement durations, respectively.

A corrective replacement is carried out when repair fails. The latter situation is assumed to occur when the repair duration exceeds a maximum value D_s.

2.3. Logistics

Several intervention teams are considered. Each intervention team can perform all the interventions on the units.

After repair, as the actual state of the component is not perfectly known, the next preventive maintenance time of this component is anticipated and scheduled a given constant time interval after the repair end.

Due to the non-zero intervention durations and the possibility of common cause failures, we can have simultaneously failed components. When an intervention is needed, if there is no maintenance team available, the intervention is postponed until a maintenance team finishes its current intervention. We introduce priority rules to know which intervention starts when several interventions are scheduled at the same time. Corrective replacements are always performed first, then repairs are considered; next, preventive replacements are given priority with respect to preventive maintenance actions.

Most companies keep a spare part inventory in order to reduce the risk of not being able to replace a failed component. Therefore our model includes a spare part management. If there is no spare part in stock at the time a replacement is scheduled, this replacement is postponed until a new delivery of spares takes place. This is very penalizing when a corrective replacement is needed.

The inventory is supplied in this way: when the stock goes below a predetermined threshold S, a quantity of Q units is ordered. The costs associated to the management of the inventory are the purchase cost of the units, the order cost, C_t, and the storage cost, c_s (modelled by a possession rate of the economic value of the units in %).

It is not simple to calculate the optimal value of Q. When there is only one type of component, the inventory is supposed to be managed by the well-known Wilson formula, modified to take into account the variability of the demand. The Economic Order Quantity [13] Q^* is thus:

$$Q^* \approx \sqrt{\frac{2C_tD}{c_s\rho}} \tag{4}$$

where D is the average demand and ρ the economic value of an item (as a first approximation, the economic value is assumed to be equal to the purchase cost). If L is the mean lead time between the replenishment order and the delivery of the component, the threshold S is equal to DL, plus, possibly, a security margin to limit the risk of stock-out.

When the manager takes the decision to replace the old-type units by new-type ones, in general, the spare part inventory is not empty. In our model two options are envisioned for the old-type spares: they can be either used to replace failed old-type components, or resold. In the same way as the K strategy was defined, the manager can make a compromise and first use a part J of the old-type spares to replace old-type components, before reselling the remaining old-type spares.

The expected lifetime of the old-type and the new-type units are not the same. Thus, each time a new-type component is installed, both the economic quantity Q and the corresponding threshold S vary. But, in the majority of cases, it is unrealistic to envisage readjusting unceasingly the spare part reapprovisioning.

In order to minimize the organizational changes, three provisioning periods are considered.

First, the old-type components are replaced by old-type spares, up to a maximum number J. Additional old-type units are no longer bought; when the remaining stock goes below a level defined by the demand on old-type components and the average lead time on new-type components, a replenishment is ordered. The remaining old-type units are then resold. In the numerical applications presented in Section 3, we assume that the resale price decreases with time as a stepwise function.

Secondly, during the transition period between the two technologies, the demand of new-type units is mostly conditioned by the threshold criterion on age that triggers the replacement of the old-type units. Indeed, the new-type components should not need frequent replacements at the beginning of their lifetime. Thus we can neglect their influence when we evaluate the average demand of spares. When the number of old-type units replaced by new-type ones is close to K, in order to apply the K strategy, an exceptional order must be posted with a sufficient quantity of items to face the grouped preventive replacement of $n-K$ old-type components.

Thirdly, after all old-type components are replaced by new-type components, the economic order quantity given by (4) can be exploited again with the demand corresponding to the new-type components.

Generally, when new-type units appear on the market, their price is very high and it decreases with time. This can motivate the manager to defer the beginning of the replacement of the old-type units by the new-type ones. In our model, a reduction factor of the purchase cost with time was thus added. By simplicity, we suppose that the purchase cost decreases by steps at predefined times. It will also influence the value of the economic quantity Q.

2.4. Constraints

The aim of the company is to achieve the replacement of all the old-type units by the new-type ones for a minimal cost and with the highest achievable availability of the system. But the manager can face some other constraints. For example, a legal constraint could impose to replace all old-type components by new-type ones before a predefined time, with some indemnity to pay if this objective is not met.

Moreover, the manager must forecast and respect a budget. A strategy s_1, with final expected cumulated costs higher than the costs entailed by another strategy s_2, could however be preferred if the costs induced by s_1 are more predictable (hence budgeted) or more regularly distributed over the whole replacement period than the costs incurred by s_2.

Indeed, large expenses on a limited time window require more cash available and hence involve additional costs. Too much variable costs around the expected value also complicate the budget forecast and increase the risk of going beyond the budget.

2.5. Costs

To evaluate the performance of each strategy, we estimate the expected cumulated costs it entails. The costs included in the model are the following.

- *The component purchase costs*: A purchase cost per unit is accounted for. As pointed out in Section 2.3, this price decreases with time. This decrease of the price can lead the manager to defer the purchase of the new components. In practice in our model, it can lead to increase the value of parameter J. In addition to this per-unit cost, a fixed order cost is considered for any delivery.
- *The resale value*: In order to limit the costs, the old-type units still in stock can be resold when the installation of the new-type units begins. In our model, the resale value of these units is under the initial purchase cost and it is also decreasing with time.
- *The consumption costs*: Each component has an operation cost modelled by an energy consumption rate, valid for all the components of a same generation. We suppose in our model that the old-type units have a larger consumption rate than the new-type ones. In further sections, we use (η+í) for the consumption rate of the old-type units and η for that of the new-type units.
- *Intervention costs*: The intervention costs have two different parts. The first one is a mobilization cost counted each time a team starts a series of interventions (noted r). The second one corresponds to hourly costs due to labour costs.
- *The loss-of-production costs*: When a unit is stopped, it does not consume anymore, but it does not produce either. Thus, we take into account hourly costs due to the loss of production. We supposed that the scheduled stops are less costly that the non-scheduled ones due to the fact that if a stop is scheduled, a provision can be made in order to reduce its consequences.
- *The storage costs*: As said in Section 2.3, these costs are represented by a cost per unit time, expressed in percents of the unit value.
- *Time horizon*: In the case a constraint on the replacement horizon time is envisaged, the model makes it possible to take into account some indemnity due for each old-type component still operated at this horizon time.

In order to compare the costs incurred at different times, we take into account a discount rate i_r. The value of this discount rate is related to the time unit chosen.

2.6. Strategy definition

In Ref. [2], the strategies were defined by the number K of old-type components undergoing a corrective replacement before all remaining old-type components are preventively replaced in a grouped way. Here the K strategy cannot be applied just as such.

As said in Section 2.4, the old-type components are first replaced by old-type spares, up to a maximum number J. After this first step, in most of our numerical applications, K must now be understood as the number of replacements of old-type components by new-type ones that are carried out one by one, according to a criterion such as the threshold age or the failure of a repair. After these K replacements, the "grouped preventive replacements" of the remaining $n-K$ old-type components still in use are scheduled.

In Section 3.3.3, we shall adapt the strategies by defining a vector $\boldsymbol{K}$. Each component of this vector $\boldsymbol{K}$ will be a number of old-type units to be replaced preventively by new-type ones during predefined time periods.

To deal with the possible constraints cited in Section 2.4, we shall use the threshold value τ_{max} of the age of old-type components before their replacement. This value is specific to the first J replacements by old-type spares, and can be modified for the next K corrective replacements by new-type components, in order to delay or anticipate the arrival of the new-type units in the system (see Section 3.2.4 for more details). We shall also introduce a maximum replacement time horizon t_{max} in Sections 3.2.1, 3.2.2 and 3.3.2.

To summarize, a strategy is thus defined by the values of the different parameters presented or recalled here above, when they apply. The set of relevant parameters can be only the scalar K or contain the values of J, $\boldsymbol{K}$, τ_{max} during the transition period t_{max}.

3. Simulations

Based on the model described in the previous model, we developed a Matlab program in order to evaluate via MC simulation the costs incurred by the different strategies.

In this section, we first compare the results of the MC simulation with an analytical test case. Then the influence of the addition of specific assumptions and/or strategy parameters introduced above is discussed. These simulations will show the interest of calculating not only the expected costs at the end of the time window under consideration, but also the evolution of costs with time in order to envisage a budget adapted to the implementation of the selected strategy. We envisage two different sets of reliability and cost data in order to show different aspects of the results. All the MC simulations presented in this section are done with 10^4 histories and we used a pure random sampling.

3.1. Analytical test case

Our MC code was first validated by running it on an analytical test case provided in Ref. [2]. The only parameter of the strategy is in this case the value of K. In this section, we assume that intervention durations are negligible and that the intervention costs are constant. The cost of an intervention is thus the following: the intervention cost r plus a cost per preventive replacement c_p, or a cost per corrective replacement c_f.

Discounting the cost at time 0 with the discount rate i_r, we have:

- Cost of a corrective replacement at time u: $(r+c_f)\,(1+i_r)^{-u}$.
- Cost of one corrective replacement and $n-K$ simultaneous preventive replacements at time u: $(r+c_f+(n-K)\,c_p)\,(1+i_r)^{-u}$.

The numerical values of the data chosen are given in Table 1. The energy consumption rate is $\eta+\nu$ for the old-type units and η for the new-type units.

Fig. 1 compares the expected cumulative costs calculated as a function of time by the MC simulation with the analytical results provided in Ref. [2].

Here, there is a pivot time $t_0\approx 6.91$, before which the optimal solution is the fully corrective strategy $K=10$, while after t_0, the optimal solution is $K=1$. These results are in agreement with those announced in the introduction, namely that the optimal strategy always corresponds either to $K=0$, $K=1$ or $K=n$. We can see in Fig. 1 that the MC estimations are fully consistent with the analytical solution. Depending on the strategy, the calculation time is approximately 0.4 h for 10^4 stories. The standard deviation of the expected cumulated cost at the end of the simulation time is 2.95 for $K=0$, 2.86 for $K=1$ and 3.5 for $K=10$; these values thus correspond to relative standard deviations between 10% and 15%.

Table 1
Numerical value of the data

Characteristic	Value
n unit's number	10
r intervention cost	1
c_p preventive cost	0.5
c_f corrective cost	1
λ old-type failure rate	0.1
μ new-type failure rate	0.05
η new-type energy consumption rate	0.1
ν old-type additional consumption rate	0.02

Arbitrary time and cost units are used.

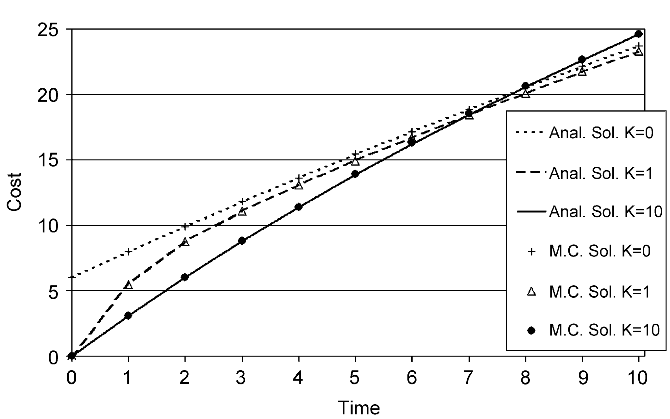

Fig. 1. Comparison between analytical results and MC simulation results.

3.2. First data set

The set of data values for these simulations are given in Tables 2 and 3.

3.2.1. Non-negligible replacement duration

The first simulation envisages a system of 100 units subject to ageing and five maintenance teams. For this first simulation, the only intervention type considered is replacement, either preventive or corrective. Therefore, the only preventive replacements

Table 2
Numerical value of the data

Characteristic	Old-type	New-type
Failure rates		
Constant term λ_0	1/420	1/420
Weibull:		
α	$300/\sqrt{\pi}$	$520/\sqrt{\pi}$
β	2	2
ν	0	0
Common mode		
χ	1/240	
$p_{O/N}$	0.65	0.60
Incompatibility		
p_0	–	0
p_i	–	0.5
f	–	1.5
Cold stand-by		
α	1/540	
β	$720/\sqrt{\pi}$	
ν	0	
Replacement time		
D_{rmin}	5/720	3/720
D_{rmax}	12/720	10/720
Erlang		
ρ	360	
γ	2	
Preventive maintenance		
Period	20	36
Effect[a]		
ε_m if $0\leqslant\tau<100$	0.75–0.95	
ε_m if $100\leqslant\tau<200$	0.8–1	
ε_m if $200\leqslant\tau$	0.89–1.09	
Preventive maintenance time		
D_{rmin}	0.5	
D_{rmax}	8/720	
Erlang		
ρ	360	
γ	2	
Repair time:		
D_{Smin}	1/720	
D_S	8/720	
Erlang		
ρ	360	
γ	2	
Repair effect[a]		
ε_r	0.9–1.1	

[a] Uniformly distributed between the two values.

Table 3
Numerical values of the costs

Cost type		Cost type	
Mobilization	300	Labor cost	10*720
Loss of production			
Scheduled	5*720	Non-scheduled	10*720
Consumption rate			
New-type	0.045*720	Old-type	0.065*720
Discount rate	0.025/12	Possession rate	0.10/12
Order cost	250	Purchase cost	1750/0.675

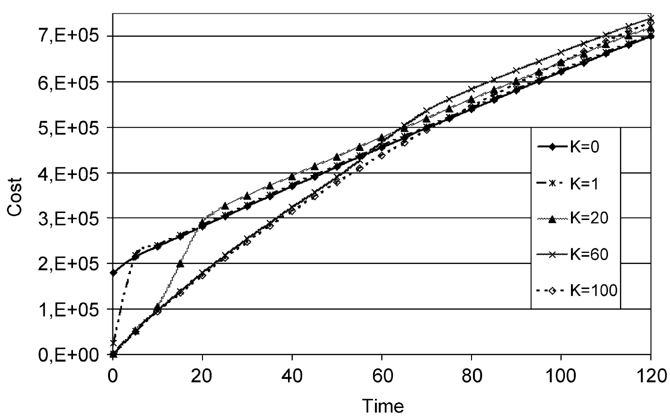

Fig. 2. Expected cumulative cost for different values of K, with only replacements and non-negligible replacement times.

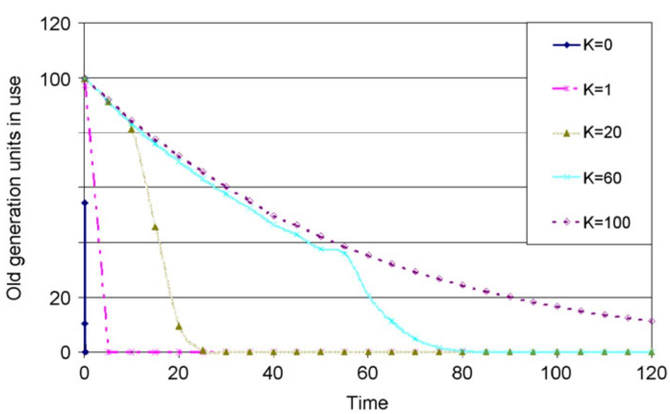

Fig. 3. Average number of old-type units running.

to be carried out are those realized in the $n-K$ grouped preventive replacements.

Here parameter J and the threshold values on age defining the replacement criteria are not considered in the different strategies because we do not envision either a spare part inventory (each component is delivered with a negligible time and a null order cost).

The only parameter of the strategies is thus parameter K, such as in Refs. [2,3].

Depending on the strategies the calculation time is about 3.5 h for 10^4 stories.

Figs. 2 and 3 show respectively the expected cumulative costs incurred for different values of K and the average number of old-type units running. With our choice of data set, the optimal strategy is the fully corrective strategy $K = 100$ until time $t \approx 74.1$; after this point, the optimal strategy is the fully preventive strategy $K = 0$. It means that this time corresponds to the trade-off between new investment costs and the higher performances of the new-type units. The standard deviation in the final cumulated cost varies from 7.02×10^4 for $K = 100$ to 9.35×10^4 for $K = 1$.

We can see in Fig. 3 that, for strategy $K = 100$, the average number of old-type units still in operation at this time is approximately equal to 27, whereas, for strategy $K = 60$, this number is equal to 2. We thus see that, if the manager wishes the replacements of the old-type units by the new ones to be completed before a given time, a maximum time horizon to complete the change of technology must be considered. Fig. 4 shows how the decision to engage the strategies within $t_s = 60$, whatever number of preventive replacements is carried out, affects the costs induced by the different strategies. In this case, we take into account a penalty cost for the remaining old-type units still in use at $t_{max} = 61$. We see that, logically, this approach disadvantages the most corrective strategies (i.e. those with large values of K). The standard deviation on the final cumulated cost varies from 2.1×10^4 for $K = 0$ to 4.4×10^4 for $K = 20$.

We can see in the figures that, at early times, the costs induced by all strategies with $K>0$ are almost the same ones. Progressively, when the probability of having already replaced K old-type units increases, each strategy leaves the bundle of curves, as K increases. When the probability to have already replaced all the old-type units is close to one, the cost evolutions associated to the different strategies become almost parallel, the system being then composed mainly of new-generation units in all cases. These asymptotically parallel evolutions appear to be more separated for other numerical values of the data set than in the case here above.

Beside these general obvious trends, a major output of this work consists in not only looking at the mean costs of each strategy at the end of the simulation time, but also in considering how regularly the costs are distributed with time. Indeed, from a budget standpoint, it can be more comfortable to deal with smoothly distributed costs instead of concentrated expenses about some specific moments.

3.2.2. *Repair and imperfect preventive maintenance*

The next simulation was performed on the full-scale problem, with the four possible types of interventions of the maintenance teams, i.e. imperfect preventive maintenance, preventive replacement, minimum repair, and corrective replacement.

In this section, we consider that there are no old-type spares at the initial moment. The stock of new components is then constituted along the lines described in Section 2.3. Namely, in a first stage, the demand of new-type units is mainly due to the replacement of the old-type units; when the number of new-type

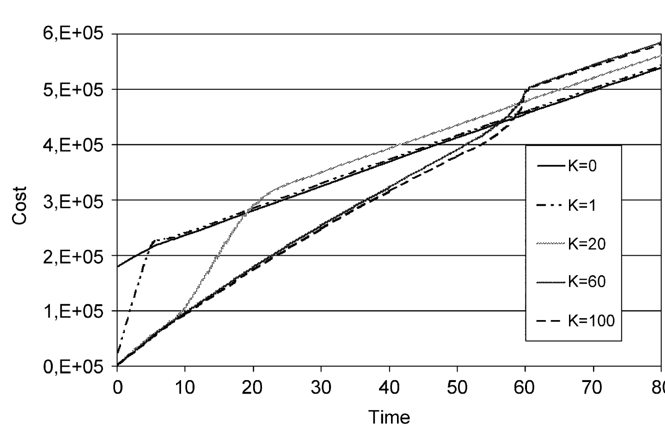

Fig. 4. Effect of the maximum time horizon on the costs.

units in use is close to K, $n-K$ units are ordered to face the demand of the $n-K$ grouped preventive replacements. After this, the demand is mainly due to failures of the new-type units already in use.

Fig. 5 shows the expected cumulated costs for the different strategies without considering a maximum time horizon for the transition between technologies, while Fig. 6 shows these costs with such a time horizon. Depending on the strategies, the calculation time is about 7 h for 10^4 stories. The standard deviation on the final cumulated cost varies from 2.7×10^4 for $K=100$ to 4.35×10^4 for $K=60$ in the case of Fig. 5. In the case of a time horizon, the standard deviation in the final cumulated cost varies from 3.3×10^4 for $K=0$ to 2.66×10^4 for $K=60$.

As we can see in Fig. 5, strategy $K=100$ is now optimal from $t\approx91$ on. This is due to the fact that the possibility of repairing or renovating the components delays the apparition of the new-type units for large values of K, and thus the costs incurred by these strategies. At this time, there is in average 44 remaining old-type units still running for strategy $K=100$.

Fig. 6 shows the costs when a maximum time horizon $t_s=60$ for the full implementation of the strategies is accounted for. With this time horizon and our set of data values, there is no visible difference between the curves corresponding to values of $K>60$. This figure shows the cost evolution until $t=80$ only, because, once all the units are new-type ones, the costs are almost the same, no matter what strategy is considered (differences being due to the age of the units and to the planning of their preventive maintenance, these differences decreasing with time).

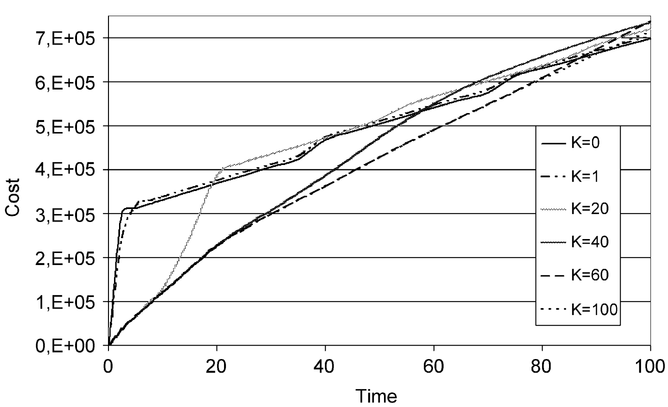

Fig. 5. Mean cumulative cost for different values of K, with the four possible interventions.

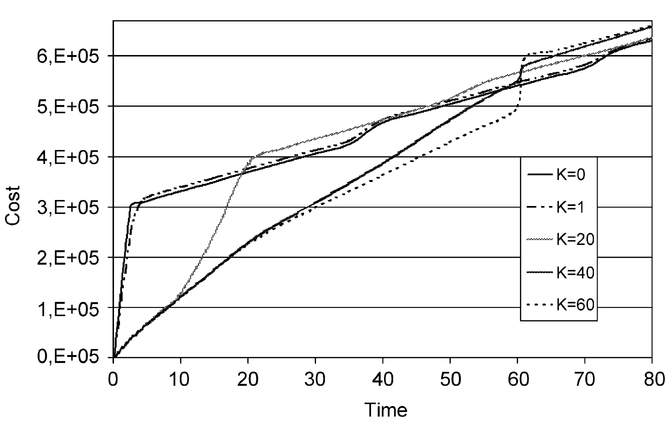

Fig. 6. Cumulative mean cost for different values of K with time horizon and the four possible interventions.

The different strategies lead to cumulative mean costs of the same order of magnitude at the final mission time; but during the application of each strategy, the costs evolve with time in a very different way according to the strategy. This implies that the optimal strategy from the total cost point of view is not automatically the most interesting strategy for the company.

Indeed, the strategy with the minimum cost at the end is strategy $K=0$; this strategy leads to a mean cost larger by several orders of magnitude than that induced by other strategies at the beginning of the strategy implementation, and up to the time where preventive replacements of old-type units by new-type ones are concentrated.

This concentration of the replacements leads also to a concentration of the preventive maintenance actions, which results in major steps in the cost curves.

We can see in Fig. 6 that strategy $K=40$ gives an almost linear evolution of the cost, which is thus more regular than for the other strategies displayed in this figure. The cost due to this strategy is thus more regularly distributed with time than those induced by the other strategies. This can lead to preferring this case with respect to another strategy entailing a smaller final total cost, as it allows to better distribute the incurred expenses on several financial years.

3.2.3. *Spare part inventory*

In our model, the parameter describing the number of old-type spare parts used is denoted by J (see Section 2.3). The manager can thus decide to use J old-type units that are still in the stock and to resell the other ones. As we said before, in this period, the stock level S when the new-type units are ordered is defined by the demand on old-type units and the delivery time of the new-type units. After the first J replacements by old-type units, the ordering of new-type spares is carried out in the same way as described in the previous section, and the remaining old-type units in stock are resold. Their reselling cost decreases with time. The values of the data related to the spare parts inventory are given in Table 4.

Fig. 7 shows the total mean costs for different values of K and J with an initial stock of 12 old-type spares. Depending on the strategies, the calculation time is about 7 h for 10^4 stories.

With our set of data, parameter J does not affect significantly the different strategies, especially for small values of K, when the advantages of the new-type units are such that it is finally more interesting not to use the old-type spares and to switch directly to new-type ones. This is partly due to our modelling of the incompatibility probability which accounts only for the number of replacements carried out and not for the time elapsed since the new-type units are available. Accounting for this last point would support strategies revealing the new-type units later.

It should be noted that, with a maximum time horizon to complete the transition period between the generations of units, the introduction of a large value of J increases the probability to

Table 4
Spare part inventory data

Characteristic	Value
Lead time:	
D_{rmin}	0.5
D_{rmax}	1.5
Erlang	
ρ	10
γ	2
Cold stand-by	
α	1/540
β	$720/\sqrt{\pi}$
ν	0

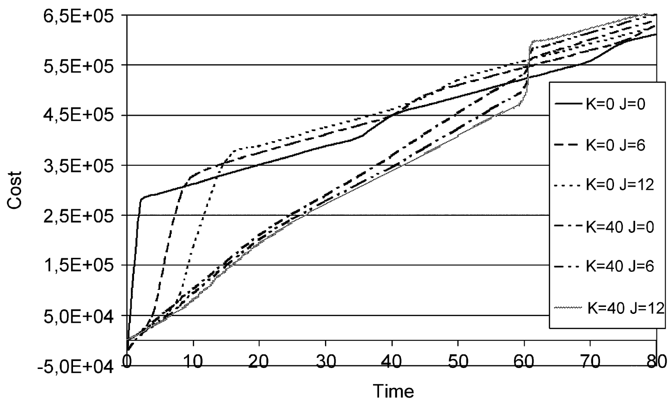

Fig. 7. Cumulative mean cost for several values of K and J.

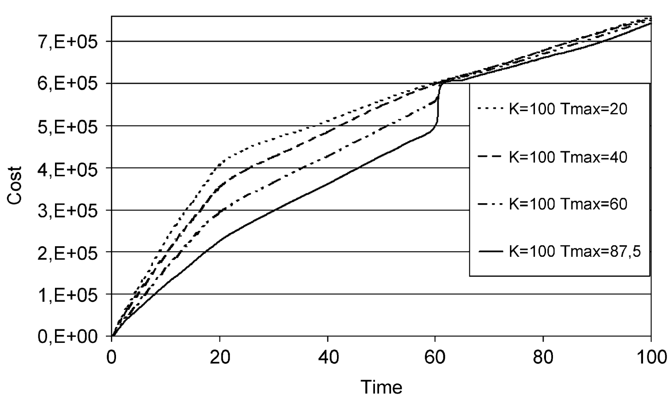

Fig. 8. Cumulative mean cost for several values of K and τ_{max}.

concentrate the preventive replacements close to this maximum horizon and thus to entail a rapid increase in the costs at this value of the time horizon, as we can observe in the curves for $K = 40$.

3.2.4. Maximum age

The last parameter that we can exploit is the value of the age threshold τ_{max} (see Section 2.1) above which old-type components are to be replaced. Indeed if, when there is no obsolescence, the threshold value of the effective age of an old component is fixed by economic considerations, it can be decreased for the transition period between the old- and the new-type units in order to anticipate the arrival of the new-type units in the system. Depending on the strategies, the calculation time is about 7 h for 10^4 stories.

Fig. 8 shows in this case the mean total costs for several values of K and τ_{max}. If the decrease of τ_{max} does not turn out to be interesting for small values of K, because it does not significantly affect the arrival time of the new-type components, we can see that, for $K = 60$, this modification of results can be used to decrease the final mean cost and to better distribute costs on the mission time compared to the same case in Fig. 6 (with $\tau_{max} = 87.5$). As seen in this figure, there is an optimal value of τ_{max}: if we decrease too much the value of τ_{max}, we concentrate the cost about the early times (see curve with $\tau_{max} = 20$) and we increase the total mean cost. The standard deviation in the final cumulated cost varies from 2.66×10^4 for $\tau_{max} = 87.5$ to 2.66×10^4 for $\tau_{max} = 20$.

3.3. Second data set

As the previous simulations show, the average cumulated cost is not the only relevant information and we envisaged in Ref. [10] a new set of data for which not only the costs are evaluated but also the standard deviations. Tables 5 and 6 give the data values used for these simulations. Here the number of units n is equal to 20. The calculation time is, depending on the strategy, approximately 8 h for 10^4 stories.

3.3.1. No maximum time horizon

The first simulation gives the costs incurred by a simple K strategy, without time horizon, as a reference result. The other results are displayed in Fig. 9. Fig. 10 shows the corresponding standard deviations. The step displayed in the costs at time $t \approx 6$ in Fig. 9 is due to the first arrival of new-type units into the stock.

Table 5
Numerical value of the data

Characteristic	Old-type	New-type
Failure rates		
Constant term λ_0	1/120	1/210
Weibull		
α	$120/\sqrt{\pi}$	$144/\sqrt{\pi}$
β	2	2
ν	0	0
Common mode		
χ	1/240	
$p_{O/N}$	*0.6*	0.65
Incompatibility		
p_0	–	0
p_i	–	0.5
f	–	1.5
Cold stand-by		
α	1/540	
β	$720/\sqrt{\pi}$	
ν	0	
Replacement time		
D_{rmin}	5/720	3/720
D_{rmax}	12/720	10/720
Erlang		
ρ	360	
γ	2	
Preventive maintenance		
Period	20	36
Effect[a]		
ε_m if $0 \leqslant \tau < 45$	0.50–0.0.7	
ε_m if $100 \leqslant \tau < 75$	0.55–0.75	
ε_m if $75 \leqslant \tau$	0.81–1.01	
Preventive maintenance time		
D_{rmin}	0.5	
D_{rmax}	8/720	
Erlang		
ρ	360	
γ	*2*	
Repair time		
D_{Smin}	1/720	
D_S	8/720	
Erlang		
ρ	360	
γ	2	
Repair effect[a]	0.9–1.1	
ε_r		
Lead time		
D_{rmin}	0.5	
D_{rmax}	1.5	
Erlang		
ρ	10	
γ	2	

[a] Uniformly distributed between the two values.

As in the previous case, if the strategy $K=n$ has the smallest cost until a time $t\approx 49.5$, on average, it does not lead to a full replacement of all the old-type units by the end of the simulation time. And at the end of the simulation time, the curves for each strategy are almost parallel and have the same magnitude order.

But, as we can see in Fig. 10, their standard deviations are quite different. Most curves display two peaks. The first one is due to the first command of new-type components. The second one is due to the grouped replacement in the K strategy. The higher the value of K, the more the peak is spread out and the lower it is. This is due to the fact that, when K increases, a smaller number of preventive grouped replacements is needed but the start time of these grouped replacements can be, from one history to another, distributed in a larger interval. The strategies with a large value of K will thus have the disadvantage to not make it possible to envisage a reasonable budget as a function of time with high precision. On the contrary, for strategies with a low value of K, if the probability of having to carry out the grouped preventive replacements is tightened much in time, these strategies present the drawback to require much more money at a given moment.

Let us note that for the $K=0$ strategy, the two peaks are superposed, and that for the $K=n$ strategy the second peak is non-existent because all the components of the old generation are replaced progressively.

Table 6
Numerical values of the costs

Cost type		Cost type	
Mobilization	300	Labor cost	10*720
Loss of production:			
Scheduled	4*720	Non-scheduled	40*720
Consumption rate:			
New-type	0.045*720	Old-type	0.065*720
Discount rate	0.025/12	Possession rate	0.10/12
Order cost	250	Purchase cost	1750/0.675

3.3.2. Time horizon

The large values of K have the advantage to spread out expenses in time. But in turn they do not guarantee to replace all

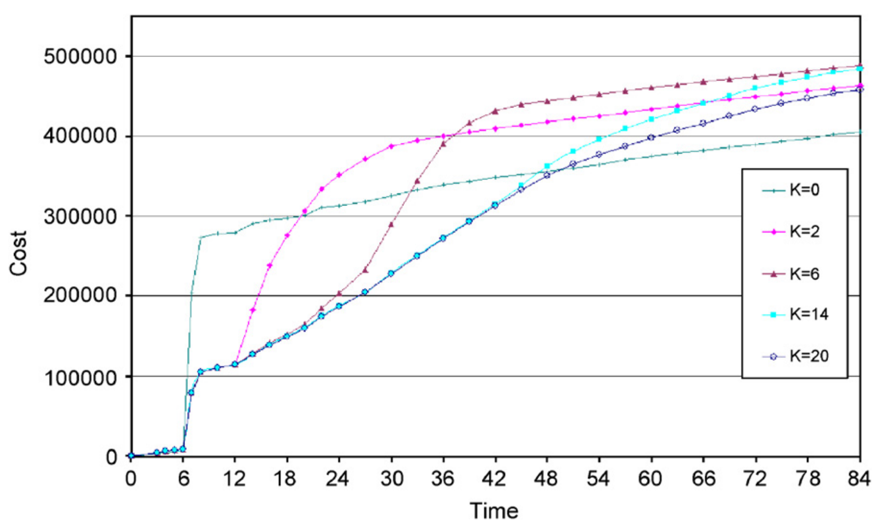

Fig. 9. Expected cumulative cost for different values of K, with only replacements and non-negligible replacement times.

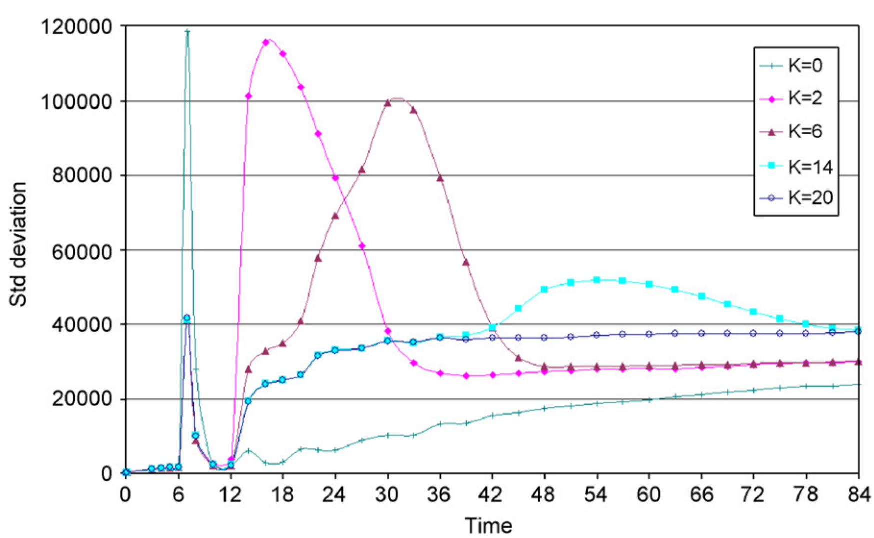

Fig. 10. Standard deviation for several values of K.

the old-type units in a reasonable time and decrease the precision of the expenditure forecast.

In Sections 3.2.1 and 3.2.2, we had considered a first approach to solve the problem of a time horizon before which all the old-type units had to be replaced. The approach was to introduce a time t_s for which the strategies are engaged, whatever number of preventive replacements is carried out. This approach has the disadvantage to concentrate again the cost around this limit time, which is precisely what we wanted to avoid. Another approach is to envisage a given time t_e before the time horizon t_{max} and to evaluate, at regular time intervals between t_e and t_{max}, if, in the current state of the system, the average replacement rate is sufficient to engage the strategy before t_{max}. If not, a small number of old-type units are preventively replaced to accelerate their global replacement. This number is selected equal to the ratio of old-type units still running over the number of remaining intervals before t_{max}.

Fig. 11 compares the average number of components of new generation in service as a function of time in two cases: for strategies without a maximum time horizon of replacement and for strategies with a maximum time horizon $t_{max} = 60$. The time t_e is chosen equal to 54, and the interval of time between each evaluation is 1.5. These strategies are applied for values of K equal to 10, 14 and 20.

We can see, in Fig. 11, that these strategies make possible to increase progressively the number of new-type units running in the system for large values of K. But all the old-type units are not yet replaced by new-type ones in t_{max}. In Fig. 12, the costs of these strategies are displayed. The strategies with t_e and t_{max} induce a lighter concentration of the costs than those using only t_s such as in Section 3.2.2. And at the end of the simulation time, these strategies lead to similar expected cumulated costs as the strategies without t_{max}. If they do not lead to an excessive jump in the costs, they display standard deviations of the same magnitude order and thus they do not improve the forecast of a budget.

3.3.3. Expenditure threshold

In order to decrease the variability of the costs, we introduce a first model of budget management. For each financial period, if the incurred costs exceed a given threshold, all the preventive interventions are postponed to the next period. Only the corrective interventions will be made in the remaining of the period. Of course with this strategy, small values of K have no

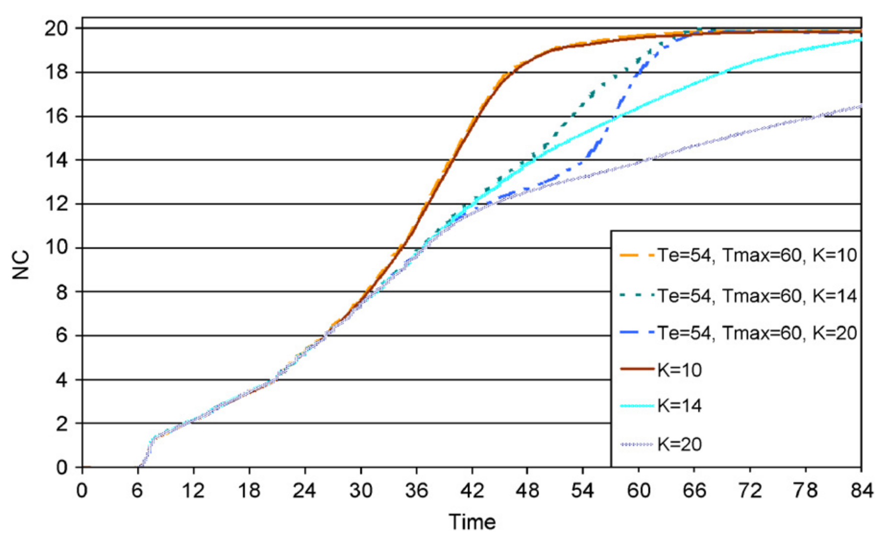

Fig. 11. Expected number of new-type units running in time for several values of K.

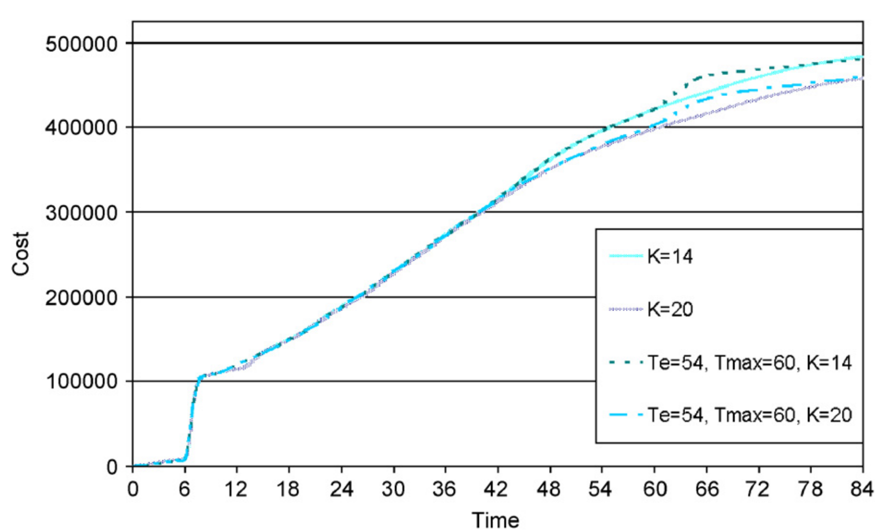

Fig. 12. Comparison of the average costs cumulated for strategies with and without horizon time.

meaning anymore. Indeed, the corresponding strategies need a large number of preventive replacements in a small period of time and they will inevitably exceed the threshold cost.

To replace all the old-type units in a reasonable time and without stopping all the preventive maintenance actions, we have to replace only a small number of old-type units by new-type ones in each financial period. This leads to reinterpreting parameter K defining the replacement strategy, and to replacing it by a vector (as mentioned in Section 2.6). Each component of this vector $\boldsymbol{K}$ is then a number of old-type units to replace preventively by new-type ones at predefined times.

Fig. 13 gives the costs incurred by different strategies of this type. The predefined times are the beginning of each financial period. Each financial period has a duration equal to 12 time units. The expenditure threshold was chosen equal to 72×10^3, the final cumulative expected costs of strategy $K=0$ divided by the number of financial periods at the end of the simulation. For the sake of comparison, the curves for $K=0$ and 20 from Fig. 9 are also showed.

Vector $\boldsymbol{K}$ was selected in order to achieve the replacement of the old-type units before the time $t_{\max}$ defined in Section 3.3.2.

We can see that these strategies do not show significant differences between them. Compared to strategy $K=0$, they induce a small overcost. It is due to the report of preventive actions, which increases the number of corrective interventions, the latter being more expensive than the preventive ones. On the other hand, the costs of these strategies $\boldsymbol{K}$ have a much more linear evolution in time than strategy $K=0$ and also slightly more linear than strategy $K=n$. The costs are thus more smoothly spread in time.

Another interesting point is that the standard deviations for these strategies, presented in Fig. 14, are lower by many orders of magnitude than those of the "traditional" K strategies, presented in Fig. 10. The unusual evolution in time of these standard deviations can be explained in the following way: the peaks are due to the grouped preventive replacements realized at the beginning of the financial periods; and the valleys are due to the limitation of the expenditures at the end of the financial periods. Of course, this reduction of the standard deviation facilitates the budget forecast.

4. Conclusions

Technological obsolescence affects most components of industrial installations, but it is rarely investigated while devising

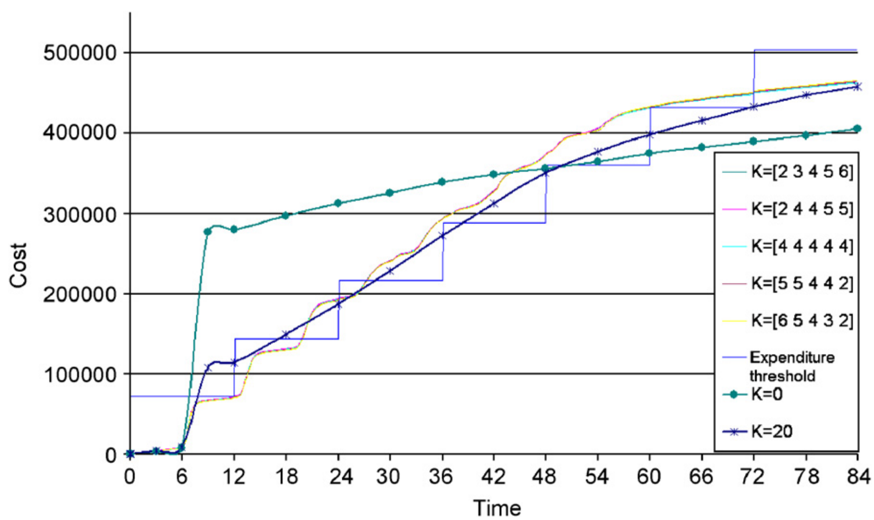

Fig. 13. Expected cumulative cost for the strategies with cost threshold.

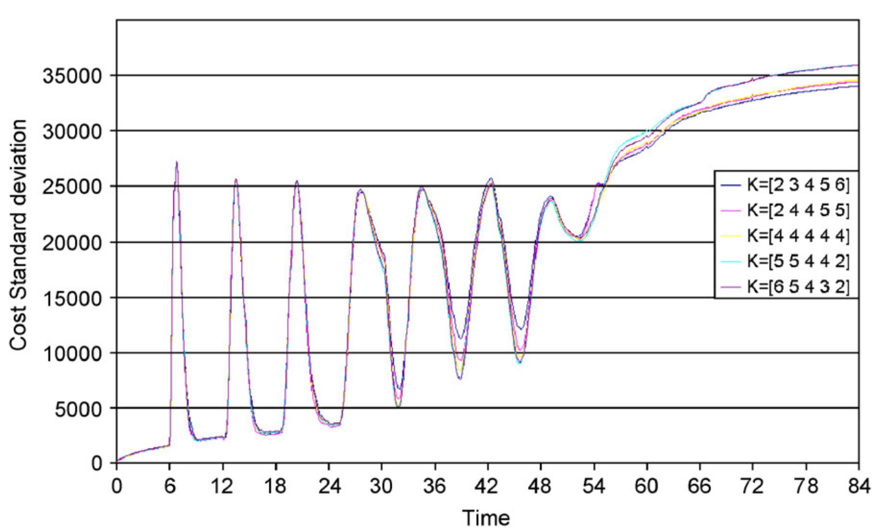

Fig. 14. Standard deviation for the strategies with cost threshold.

a maintenance strategy. The first step of this work was to develop a model dealing with maintenance strategies authorizing different types of preventive and corrective interventions. The proposed model is included in an MC simulation.

The MC code was used to calculate the average cumulative cost incurred by the replacement strategies as a function of time, be a strategy fully completed or not. We envisaged not only the expected costs but also their standard deviations. This information is important to choose a maintenance and replacement strategy. Indeed, strategies with lower final costs can also display jumps in the costs at some times or a much larger statistical variability than apparently less performing strategies. In this case, strategies inducing lower final costs could exceed with a significant probability the budget on the corresponding financial period, which would negatively affect the actual expected costs.

The results, first, show the impact of each strategy parameter on the costs induced by the different strategies studied. This work, in particular, made it possible to take into account the impact of the decisions relative to the use of the old-type spare parts. It also made possible to give a first modelling of the strategy definition for the build-up of new-type units' spare parts inventory.

The opportunity to speed up the replacement of the old-type units before the end of their lifetime by adapting the replacement criterion (i.e. in this work by decreasing the value of the replacement age threshold $\tau_{\max}$) was also discussed.

Finally, we proposed strategies to deal with the constraint of a time horizon before which all the old-type units have to be replaced. We envisaged also strategies allowing to reducing the variability of the costs.

There are several perspectives to improve the proposed model. The model could have to deal with more constraints, like e.g. non-negligible replacement durations. We will also refine the obsolescence modelling, for example, by taking into account the possible availability on the market of successive generations of challenger units, or, by considering the possibility of choosing between different available challenger unit types. Another way of refining the model is to consider not only identical units but a more complex system with several different units. In this case, the stock model should be adapted consequently.

References

[1] Borgonovo E, Marseguerra M, Zio E. A Monte Carlo methodological approach to plant availability modeling with maintenance, aging and obsolescence. Rel Eng Syst Safety 2000;67:61–73.

[2] Mercier S, Labeau PE. Optimal replacement policy for a series system with obsolescence. Appl Stochast Model Bus Ind 2004;20:73–91.

[3] Elmakis D, Levitin G, Lisnianski A. Optimal scheduling for replacement of power system equipment with new-type ones. In: Proceedings of MMR'2002, Trondheim, Norway, 2002. p. 227–30.

[4] Michel O, Labeau PE, Mercier S. Monte Carlo optimization of the replacement strategy of components subject to technological obsolescence. In: Proceedings of PSAM 7—ESREL'04. Berlin, Germany, 2004. p. 3098–103.

[5] Clavareau J, Labeau PE. Maintenance and replacement policies under technological obsolescence. In: Proceedings of ESREL'06. Estoril, Portugal, 2006. p. 499–506.

[6] Clavareau J, Labeau PE. Maintenance et stratégies de remplacement de composants soumis à obsolescence technologique. In: Proceedings of lambda mu 15, Lille (France), electronic version: LM15-COM-114.pdf, 2006 [in French].

[7] Villemeur A. Sûreté de fonctionnement des systèmes industriels. Editions Eyrolles, Paris, 1988 [in French].

[8] Wu S, Clements-Croome D. Preventive maintenance models with random maintenance quality. Rel Eng Syst Safety 2005;90:99–105.

[9] Fouathia O, Maun JC, Labeau PE, Wiot D. Cost-optimization model for the planning of the renewal, inspection, and maintenance of substation facilities in Belgian power transmission system. In: Proceedings of ESREL 2005, Gdansk, Poland, 2005. p. 631–7.

[10] Kijima M, Morimura H, Suzuki Y. Periodical replacement problem without assuming minimal repair. Eur J Oper Res 1988;37:194–203.

[11] Doyen L, Gaudoin O. Classes of imperfect repair models based on reduction of failure intensity or virtual age. Rel Eng Syst Safety 2004;84:45–56.

[12] Sanchez A, Carlos S, Martorell S, Villanueva FJ. Addressing imperfect maintenance modelling uncertainty in unavailability and cost based optimization. Rel Eng Syst Safety 2007.

[13] van Delft Ch, Vial JP. Discounted costs, obsolescence and planned stockouts with the EOQ formula. Int J Prod Econ 1996;44:255–65.